Comment on "Effect of a polarized hydrogen target on the polarization of a stored proton beam"

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Meyer [Phys. Rev. E **50**, 1485 (1994)] analyzed the filtering mechanism of polarizing a stored beam by scattering from an internal polarized target. We noticed in Meyer's derivation of Eq. (4) of that paper that he had added a new twist to an old argument [W. Brückner *et al.*, Physics with Antiprotons at LEAR in the ACOL Era: Proceedings of the Third LEAR Workshop, Tignes, Savoie, France, January 19–26, 1985 (Editions Frontières, Gif-sur-Yvette, France, 1985), p. 245] by allowing some particles that are spin flipped to be kept in the beam. We show that this invalidates the old result and leads to a more complicated expression for the buildup of polarization.

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In his paper [2], Meyer performed a more critical analysis of the theory behind polarization filtering [3] with respect to understanding the deviation of the results of a previous experiment [4] from the theoretical expectation [1]. Meyer gives the polarization rate [Eq. (2) of Ref. [2]]

$$\frac{dP_B}{dt} = (1 - P_B^2)\frac{1}{2}\left(\frac{1}{N_\uparrow}\frac{dN_\uparrow}{dt} - \frac{1}{N_\downarrow}\frac{dN_\downarrow}{dt}\right), \tag{1}$$

which he obtained by differentiating the common definition of the beam's polarization,

$$P_B = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}, \tag{2}$$

with respect to time t where $N_\uparrow$ and $N_\downarrow$ are the number of beam particles in the spin-up and -down states, respectively. He also states that the derivatives on the right side of Eq. (1) can be nonzero due to both losses and spin flipping. He then invokes some proportionality arguments, i.e., that the resultant polarization scales linearly with the target polarization P_T, target thickness d, and revolution frequency f_R. He says that what remains is a "constant of proportionality" $\hat{\sigma}$ that is essentially a "polarizing cross section." From this argument he then obtains the differential polarization rate in his Eq. (3) of Ref. [2]:

$$\frac{dP_B}{dt} = (1 - P_B^2) f_R d P_T \hat{\sigma}. \tag{3}$$

This equation is sound for the case of no spin flipping, since in that case neither of the fractional rate derivatives $(1/N_\uparrow)dN_\uparrow/dt$ and $(1/N_\downarrow)dN_\downarrow/dt$ involve the other spin state, and so the terms inside the large parentheses of Eq. (1) would depend on the above scaling parameters only as indicated in Eq. (3). Without spin flipping, each equation for decay rate of the number of particles in a given spin state is independent of the other state.

However, spin flipping is a mechanism that moves particles from one spin state to the other spin state, so now there is a coupling between the spin-up and spin-down decay rates. For simplicity, let us assume that the target is fully polarized ($P_T=1$) with any other target parameters such as thickness and density held constant. Also assume that the beam's revolution frequency is constant. Differential equations for these rates of both spin states may be written in the form

$$\frac{d}{dt}\begin{pmatrix} N_\uparrow \\ N_\downarrow \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} N_\uparrow \\ N_\downarrow \end{pmatrix} \tag{4}$$

where a and d are negative constants determined by particle loss from non-spin-flip processes and spin-flip transitions resulting in momenta outside the acceptance of the accelerator's transverse and longitudinal momentum apertures. The positive constants b and c are due to an increase in numbers from the opposite state.

The matrix

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{5}$$

has eigenvalues

$$\lambda_\pm = \frac{a+d}{2} \pm \sqrt{\left(\frac{a-d}{2}\right)^2 + bc} \tag{6}$$

corresponding to the unnormalized eigenvectors

$$v_+ = \begin{pmatrix} b \\ \lambda_+ - a \end{pmatrix}, \quad v_- = \begin{pmatrix} \lambda_- - d \\ c \end{pmatrix}, \tag{7}$$

at least in nondegenerate cases. Diagonalizing the matrix M yields

$$M' = R^{-1}MR = \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix}, \tag{8}$$

where

$$R = \begin{pmatrix} b & \lambda_- - d \\ \lambda_+ - a & c \end{pmatrix}. \tag{9}$$

By expanding $N_\uparrow$ and $N_\downarrow$

$$\begin{pmatrix} N_\uparrow \\ N_\downarrow \end{pmatrix} = C_1 v_+ + C_2 v_-, \tag{10}$$

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we can transform Eq. (4) into

$$\frac{d}{dt}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \end{pmatrix}. \qquad (11)$$

This is easily solved, giving

$$\begin{pmatrix} C_1(t) \\ C_2(t) \end{pmatrix} = \begin{pmatrix} C_1(0)e^{\lambda_+ t} \\ C_2(0)e^{\lambda_- t} \end{pmatrix}, \qquad (12)$$

where the $C_j(0)$ are the initial values at $t=0$. Multiplying by R to transform back to the original coordinates we have

$$\begin{pmatrix} N_\uparrow(t) \\ N_\downarrow(t) \end{pmatrix} = \begin{pmatrix} bC_1(0)e^{\lambda_+ t} + (\lambda_- - d)C_2(0)e^{\lambda_- t} \\ (\lambda_+ - a)C_1(0)e^{\lambda_+ t} + cC_2(0)e^{\lambda_- t} \end{pmatrix}. \qquad (13)$$

Solving for $C_1(0)$ and $C_2(0)$ in terms of initial conditions $N_\uparrow(0)$ and $N_\downarrow(0)$ produces

$$\begin{pmatrix} C_1(0) \\ C_2(0) \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} cN_\uparrow(0) + (d-\lambda_-)N_\downarrow(0) \\ (a-\lambda_+)N_\uparrow(0) + bN_\downarrow(0) \end{pmatrix}, \qquad (14)$$

where Δ is the determinant of R. Substituting these results into the previous equation yields

$$\begin{pmatrix} N_\uparrow(t) \\ N_\downarrow(t) \end{pmatrix} = \frac{1}{\Delta}\begin{pmatrix} b[cN_\uparrow(0) + (d-\lambda_-)N_\downarrow(0)]e^{\lambda_+ t} + (\lambda_- - d)[(a-\lambda_+)N_\uparrow(0) + bN_\downarrow(0)]e^{\lambda_- t} \\ (\lambda_+ - a)[cN_\uparrow(0) + (d-\lambda_-)N_\downarrow(0)]e^{\lambda_+ t} + c[(a-\lambda_+)N_\uparrow(0) + bN_\downarrow(0)]e^{\lambda_- t} \end{pmatrix}. \qquad (15)$$

The polarization of the beam is

$$P_B(t) = \frac{N_\uparrow(t) - N_\downarrow(t)}{N_\uparrow(t) + N_\downarrow(t)} = \frac{(b+a-\lambda_+)[cN_\uparrow(0) + (d-\lambda_-)N_\downarrow(0)]e^{\lambda_+ t} + (\lambda_- - d - c)[(a-\lambda_+)N_\uparrow(0) + bN_\downarrow(0)]e^{\lambda_- t}}{(b-a+\lambda_+)[cN_\uparrow(0) + (d-\lambda_-)N_\downarrow(0)]e^{\lambda_+ t} + (\lambda_- - d + c)[(a-\lambda_+)N_\uparrow(0) + bN_\downarrow(0)]e^{\lambda_- t}}, \qquad (16)$$

or if we assume an initial polarization of zero with $N_\uparrow(0)=N_\downarrow(0)$ then

$$P_B(t) = \frac{(b+a-\lambda_+)(c+d-\lambda_-)(e^{\lambda_+ t} - e^{\lambda_- t})}{(b-a+\lambda_+)(c+d-\lambda_-)e^{\lambda_+ t} + (b+a-\lambda_+)(c-d+\lambda_-)e^{\lambda_- t}}. \qquad (17)$$

When there is no spin flipping $b=c=0$, the polarization simplifies to

$$P_B = \frac{e^{\lambda_- t} - e^{\lambda_+ t}}{e^{\lambda_- t} + e^{\lambda_+ t}} = \frac{\exp\left[-\sqrt{\left(\frac{a-d}{2}\right)^2}\,t\right] - \exp\left[\sqrt{\left(\frac{a-d}{2}\right)^2}\,t\right]}{\exp\left[-\sqrt{\left(\frac{a-d}{2}\right)^2}\,t\right] + \exp\left[\sqrt{\left(\frac{a-d}{2}\right)^2}\,t\right]} = \pm\tanh\left(\left|\frac{a-d}{2}\right|t\right). \qquad (18)$$

The sign ambiguity can be resolved by realizing that if d is more negative than a, then the scattering will yield a net positive polarization with $(a-d)/2>0$, so

$$P_B = \tanh\left(\frac{a-d}{2}t\right). \qquad (19)$$

This result agrees in the hyperbolic tangent form with the earlier non-spin-flip prediction of Ref. [1]. Equation (19) also has the same functional form as Meyer's Eq. (4) (see Ref. [2]):

$$P_B(t) = \tanh(t f_R d P_T \hat{\sigma}), \qquad (20)$$

but the functional form of Eq. (17) deviates from the form of a hyperbolic tangent dependence when spin flipping is allowed with b and c being nonzero. So we must conclude that Meyer's Eq. (4) is incorrect given that he allows some particles to remain in the beam after having their spins flipped.

Recently another report [5] appeared which gives an equivalent derivation in terms of kinetic equations and shows that the spin-flip terms are indeed very small.

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